

LAIKIPIA



STAT 213

UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

**SECOND YEAR EXAMINATION FOR THE DEGREE OF
BACHELOR OF SCIENCE (ECONOMICS & STATISTICS)**

STAT 213: PRINCIPLES OF STATISTICAL INFERENCE

STREAM: R

TIME: 2 HRS

DAY: TUESDAY [8.30 A.M – 10.30 A.M]

DATE: 09/04/2024

THIS QUESTION PAPER CONSISTS OF FIVE (5 PAGES

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INSTRUCTIONS

- (i) Answer question one and any two questions
- (ii) Show all the workings clearly
- (iii) Do not write on the question paper

QUESTION ONE (30 MARKS)

- a) Briefly explain the difference between the following terms
- i) Probability of Type I and probability of Type II errors
 - ii) Null and alternative hypotheses
 - iii) One-tailed and two tailed tests
 - iv) Simple and composite hypothesis **(8 Marks)**
- b) The amount of time that customer spends at an airport check-in counter is a normal random variable with standard deviation 1.5 minutes. Suppose that a random sample of size $n = 49$, customers with mean 8.2 minutes is observed. Find a 95% Confidence interval for the mean of waiting time. **(4 Marks)**
- c) Let $X_1, X_2, X_3, \dots, X_n$ be random sample from a geometric distribution function.

$$f(x;\theta) = \begin{cases} \theta(1 - \theta)^x & x = 0, 1, \dots, n \\ 0 & elsewhere \end{cases}$$

Find; Method of moments estimator (MME) of θ **(4 Marks)**

- d) A group of 10 college students were asked to report the number of hours that they slept on the previous night with the following results: 7.0, 6.5, 7.25, 8.5, 5.0, 8.0, 7.0, 6.75, 5.75, 7.15. Find a 90% confidence interval for the average number of hours that college students sleep. **(5 Marks)**
- e) If s_1^2 and s_2^2 represent the variances of independent random samples of size $n_1 = 6$ and $n_2 = 9$, taken from normal populations with variances $\sigma_1^2 = 15$ and $\sigma_2^2 = 10$ respectively.

Find $P\left[4.095 < \frac{s_1^2}{s_2^2} < 12.45\right]$ **(5 Marks)**



- f) Suppose we want to estimate the population mean with margin of error of 1.5 at 90% confidence interval. The population standard deviation is estimated to be 65 how large a sample is required? **(4 Marks)**

QUESTION TWO (20 MARKS)

- a) Differentiate between the following terms as used in point estimation.

Estimator and an estimate

(4 Marks)

- b) An engineer is concerned that the variability of responses using two different procedures may not be the same. Before conducting his research, he conducts a pre-study with random samples of 10 and 8 responses and gets $s_1^2 = 7.14$ and $s_2^2 = 3.21$, respectively. Do the sample variances present sufficient evidence to indicate that the population variances are unequal? Use $\alpha = 0.05$ **(6 Marks)**
- c) The mean number of travel days per year for marketers is to be estimated. Past studies indicate that the travel days are approximately normal with a standard deviation of 10 days. What sample size would have to be taken so that a person is 94% confident that the sample mean will be within 2 days of the true mean travel days of all marketers **(4 Marks)**
- d) A particular brand of diet additive was analyzed to determine the level of polyunsaturated fatty acid (in percentages). A sample of 8 packages resulted in the following data: 16.8, 17.2, 17.4, 16.2, 16.9, 17.1, 18.2, 16.5. Find a 95% prediction interval for the polyunsaturated fatty acid in the next package of margarine that is tested. **(6 Marks)**

QUESTION THREE (20 MARKS)

The wearing qualities of two types of machine rotating parts were compared by testing samples of $n_1 = n_2 = 10$ for each part. The number of hours until wear out was defined as specific amount of time of machine part wear and tear. The means and variance are $\bar{x}_1 = 26,400$, $s_1^2 = 1,440,000$, $\bar{x}_2 = 25,100$ and $s_2^2 = 1,960,000$.

- a) Find a 95% confidence interval for the difference in mean wearing quality for the two types of machine parts **(8 Marks)**
- b) Use the above confidence interval to determine whether there was a significant difference in mean wearing quality for the two types of machine parts **(4 Marks)**
- c) Construct the 99% interval for the ratio of the two population variances and interpret it **(5 Marks)**
- d) What assumptions must be made in order that the confidence intervals obtained in parts (a) and (c) are valid? **(3 Marks)**

QUESTION FOUR (20 MARKS)

- a) State the four properties of estimators. **(4 Marks)**
- b) Professor Nelson has recently employed a new gateman who is expected to open the gate of the entrance path to his home every time the business man returns from work. Over 12 working days he records the number of times he has to ring the bell until the gateman opens the gate. The records are as follows; 7,4,10,1,5,2,8,0,3,7,2,6,
Does he have good reason to replace the watchman? Note that a smart watchman respond to the third ring of the bell i.e. $\mu_0=3$ (Use $\alpha = 0.01$). **(5 marks)**
- c) Differentiate between the following terms.
Statistical hypothesis and Test of hypothesis. **(4 Marks)**
- d) State the steps for general procedure for hypothesis testing. **(7 Marks)**

QUESTION FIVE (20 MARKS)

- a) The table below shows the marks awarded to nine children in a competition.

| Child | A | B | C | D | E | F | G | H | I |
|---------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Judge 1 | 6.8 | 7.3 | 8.1 | 9.8 | 7.1 | 9.2 | 8.1 | 8.2 | 7.3 |
| Judge 2 | 7.8 | 9.4 | 7.9 | 9.6 | 8.9 | 6.9 | 7.8 | 7.9 | 9.4 |

- i. Compute the Spearman’s rank correlation coefficient for the data and comment on the value obtained. **(5 Marks)**
- ii. Using the hypotheses

$H_0: \rho=0$ no correlation exists



Against $H_1: \rho \neq 0$ correlation exists

Compute the test statistic (3 Marks)

iii. Determine whether there is any significant correlation in the voting by the readers and critics at 0.05 level of significance. (2 Marks)

b) The mean water temperature downstream from a power plant cooling tower discharge pipe should be no more than 102°F. Past experiences has indicated that the s.d of temperature is 4°F. The water temperature is measured on 9 randomly chosen days and the average temperature is chosen to be 98°F. Should the water temperature be judged acceptable with $\alpha = 0.01$ (4 Marks)

c) Let $X_1, X_2, X_3, \dots, X_n$ is a random sample from a uniform p.d.f

$$f(x;a,b) = \begin{cases} \frac{1}{a+b}, & a < x < b \\ 0, & \text{elsewhere} \end{cases}$$

Find

The Method of moments estimator (MME) of a and b (6 Marks)

